Experimental observation of two-state on-off intermittency

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We observe a two-state on-off intermittency in a diode laser with an external cavity experimentally. The control parameter is the length of the external cavity that is periodically modulated. We demonstrate that the system exhibits an intermittent behavior between a two-mode chaotic regime and a single-mode state when the control parameter passes through the bifurcation point. Power-law scaling of the average laminar time with a critical exponent of -1 is found as a function of both the amplitude and frequency of the external modulation.

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The intermittency route to chaos may be observed in a dynamical system when a control parameter passes through a critical value. The intermittent behavior is characterized by irregular bursts (turbulent phases) interrupting the nearly regular (laminar) phases. The duration of the turbulent phases is fairly regular and weakly dependent on the control parameter, but the mean duration of the laminar phases decreases as the control parameter increases beyond a critical value, and eventually they disappear. Hence only one bifurcation point is associated with the intermittency route to chaos.

Different types of intermittency have been observed in lasers: type I, type II, and type III of the Pomeau-Manneville intermittency [1], on-off [2], and crisis-induced intermittency [3]. The type of intermittency depends on the type of bifurcation at the critical point. The type I and on-off intermittency are associated with saddle-node bifurcations, the type II and type III with Hopf and inverse period-doubling bifurcations, respectively, and the crisis-induced intermittency with the crisis of chaotic attractors when two (or more) chaotic attractors simultaneously collide with a periodic orbit (or orbits) [4].

On-off intermittency differs from other types of intermittency because it requires a dynamical time-dependent forcing of a bifurcation parameter through a bifurcation point [5], whereas for other types of intermittency the parameters are fixed. This type of intermittency is often called *modulational intermittency* [6]. This name is derived from the characteristic two-state nature of the intermittent signal. One or more dynamical variables of the system exhibit two distinct states as the system evolves in time. In the ''off'' state the variables remain approximately constant in various time intervals. These periods are called laminar phases. The ''on'' states are characterized by irregular bursts of the variables away from their constant values.

In previous studies of the on-off intermittency the parameter was driven either randomly or chaotically [5,7,8]. In both cases, typical power-law scalings have been found near the onset of intermittency (i) for the mean laminar phase as a function of the coupling parameter with a critical exponent of -1 and (ii) for the probability distribution of laminar phase versus the laminar length with exponent $-3/2$ [8]. It was shown that these power laws are valid for a large class of randomly driven systems. The on-off intermittency has been also detected experimentally in a nonlinear electronic circuit tuned near a Hopf bifurcation point [9] and in a Nd-YAG (yttrium aluminum garnet) laser [10]. The critical exponent of -1 for the mean laminar phase has been proved with the laser experiments as a function of the pump current near the first laser threshold [10]. However, no external modulation was applied and therefore the reason for the spontaneous on-off behavior was elusive.

A particular case of modulational intermittency called *two-state on-off intermittency* was considered recently by Lai and Grebogi [11]. In a periodically forced Duffing oscillator, which possesses two symmetric low-dimensional invariant subspaces, they observed that a typical trajectory spends a long time near one invariant subspace, is repelled away from this subspace, then possibly is attracted to the other invariant subspace or the same subspace, temporally spending a long stretch of time there, is repelled away again, etc. Due to the presence of two invariant subspaces, there are two ''off'' states. This is the characteristic feature of the two-state on-off intermittency. This intermittent behavior differs from the conventional on-off intermittency where there is only one temporally attracting state.

In this Brief Report we demonstrate an experimental observation of a two-state on-off intermittency; the experimental system is a diode laser with two external cravities. The experimental setup is displayed in Fig. 1. The laser system is based on a semiconductor laser diode with an external cavity in Littrow configuration (Sacher Lasertechnik, TEC 100). The laser source is a commercial laser diode where one facet is antireflection coated, which suppresses the reflectivity typically below 10^{-4}. The laser is current and temperature stabilized. The external cavity is defined by a diffraction grating. The zeroth order serves as the output beam, while the first order is reflected back into the diode. This way, the cavity is built by the rear facet of the diode and the grating plane. The wavelength selectivity of the grating forces the laser to oscillate in a single longitudinal mode.

The second external cavity is formed with a plane mirror (M). The feedback strength from the mirror is adjusted with a variable neutral density filter (VF). 50% beamsplitters (BS) are used to direct the laser beam to a p-i-n photodiode (PD) of 125 MHz bandwidth (New Focus 1801) and to a wavelength meter. An optical isolator (OI) is placed in front

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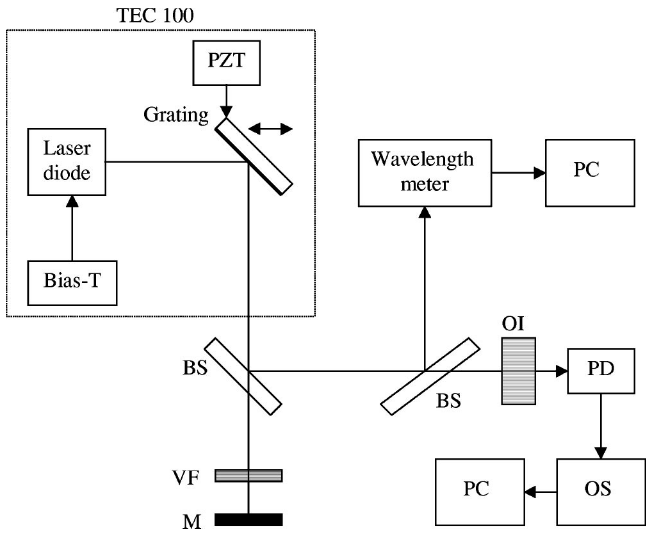

FIG. 1. Experimental setup. PZT is the piezo driver, BS are the beamsplitters, VF is the variable neutral filter, M is the plane mirror, OI is the optical isolator, PD is the photodiode, OS is the oscilloscope, and the PC's are the personal computers.

of the detector to avoid the reflection from the photodiode. The laser response is analyzed with a 200-MHz bandwidth digital oscilloscope (OS) and personal computers (PC).

When the reflection from the external mirror is cut off, the laser operates in a single-mode ($\lambda_1 = 631.505$ nm) regime [Fig. 2(a), left-hand column] that yields a steady state laser emission (right-hand column). When the second cavity is open, the second wavelength $\lambda_2 = 630.866$ nm appears, which is resonant to the cavity formed by the laser diode,

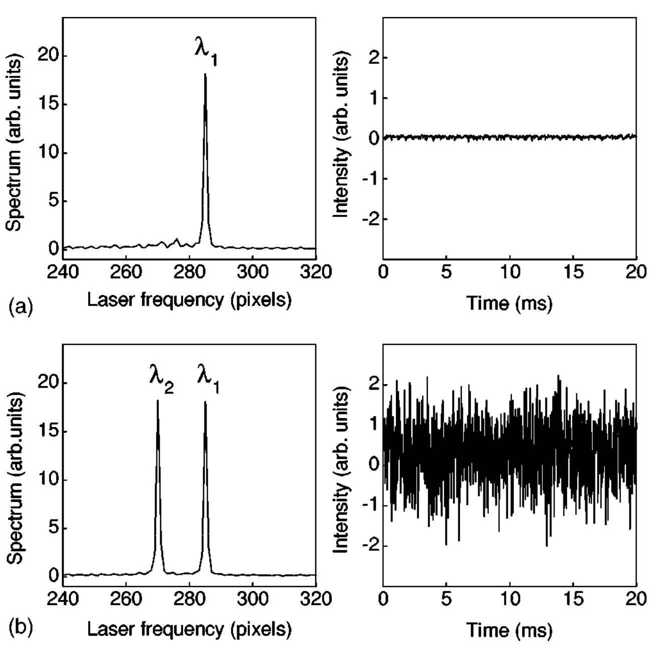

FIG. 2. Optical spectrum of laser lines (left-hand column) and corresponding time series (right-hand column) (a) with single external cavity and (b) with two external cavities. Two wavelengths $\lambda_1 = 631.505$ μm and $\lambda_2 = 630.866$ μm have approximately the same gain.

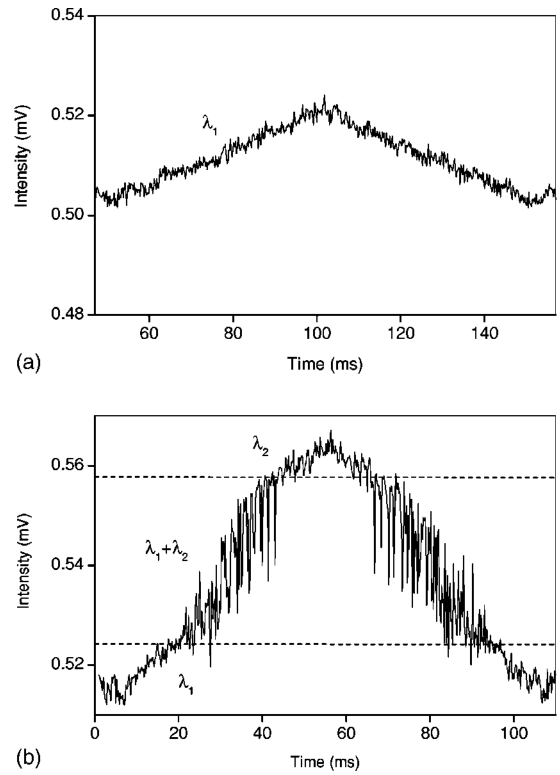

FIG. 3. Laser response to large-amplitude modulation of cavity detuning with $f = 10$ Hz. (a) The external mirror is cut off. (b) The second external cavity is open. The dashed lines indicate the onset of chaos.

grating, and mirror M. By tuning the grating with a piezo actuator (PZT) the gain factors of the laser modes can be adjusted to have approximately the same values as shown in Fig. 2(b) (left-hand column). This results in strong instabilities in the laser response [Fig. 2(b), right-hand column] with a broadband power spectrum that may be a consequence of chaos.

The origin of these instabilities can be derived from the following speculations. The spontaneous emission from a diode laser is usually concentrated into the laser modes even by the relatively low-finesse laser resonator modes. It is evident that additional resonances may channel the spontaneous energy to other frequencies. The typical resonances in such a system may be, for example, (i) the external optical cavity optical resonance frequencies, (ii) the difference of external cavity mode frequencies, (iii) the "compound" resonator (consisting of laser diode resonator plus external resonators) frequencies, and (iv) the relaxation oscillation frequency. It is intuitively plausible that, when two of these resonance frequencies match, one might expect some "resonant destabilization." Although the solitary laser is stable (as a class B laser), the reflections into the laser from the external resonators may lead to chaotic laser pulsations (see, e.g., Ref. [12] and references therein). Preliminary numerical simulations of a diode laser with the "compound" resonator [13] yield oscillatory behavior very similar to that observed in our experiments.

The transition between the two laser modes is clearly seen in Fig. 3 where we show the laser response at slow large-

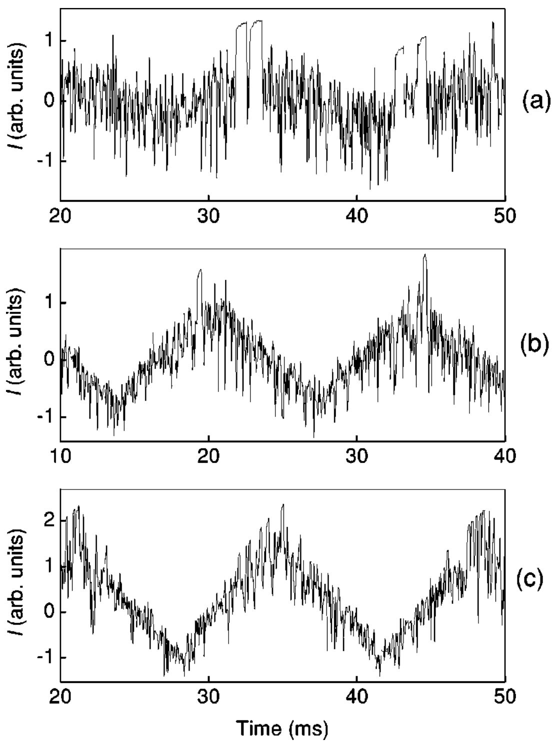

FIG. 4. Time scales of laser output in the presence of cavity detuning modulation through onset of intermittency. (a) $A=100$ mV, (b) 250 mV, and (c) 350 mV. $f=73$ Hz. Injection current $I=88$ mA. The mean laminar phase decreases with increasing A.

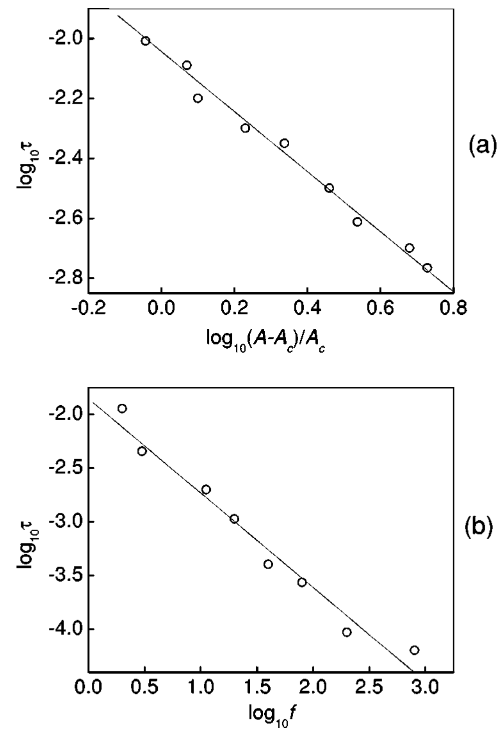

FIG. 5. Average laminar length (a) versus difference of modulation amplitude over its critical value $(A-A_c)$ for frequency $f=73$ Hz and (b) versus modulation frequency for amplitude $A=200$ mV in log-log scale. The open circles represent the measurements and the solid lines show the slopes of -1.

amplitude modulation of cavity detuning (the modulation frequency $f=10$ Hz). When the second cavity is cut off, the laser operates in a single-mode regime (λ_1) with a stable steady state emission at any value of detuning (Fig. 3(a)). Small instabilities observed in the figure result from internal noise. When the second cavity is open, the single-mode operation is observed only in the extreme positions of the grating where the laser operates in one or another mode (λ_1 or λ_2) [below and above the dashed lines in Fig. 3(b)]. The chaotic two-mode regime occurs in the range of detuning bounded by the dashed lines where the laser modes have approximately the same gain ($\lambda_1+\lambda_2$).

The external modulation is the necessary condition for the observation of the on-off intermittency. The two-state intermittency arises when the cavity detuning passes through the bifurcation point for the onset of intermittency. This bifurcation point is situated in the middle range between the dashed lines. There are two temporally attractive states: one of them is the two-mode regime (probably chaotic) and the other is the single-mode laser emission. The trajectory can spend long stretches of time near each of the two attractors that leads to the intermittent behavior. The short intervals of temporal traces of the laser output are shown in Fig. 4 for different modulation amplitudes A at fixed frequency $f=73$ Hz. Since one of the off states is chaotic, it is difficult or even impossible to distinguish this off state from the on state. However, the laminar phases corresponding to the steady state single-mode lasing are clearly distinguished in the figure.

One of the important characteristics of intermittency is the mean duration of the laminar phase. In order to characterize the intermittent behavior observed, we measure the dependence of the mean laminar phase corresponding to the single-mode emission on the amplitude and frequency of the external modulation. Without modulation ($A=0$) the laser operates in the two-mode chaotic regime [Fig. 2(b)]. With the increase of A above some critical value A_c the long laminar phases corresponding to the single-mode emission appear [Fig. 4(a)]. In these phases the gain factor for the radiation on λ_1 is smaller than for λ_2, and the latter wavelength wins in the mode competition. However, the latter state is unstable since another laser mode still exists. Higher values of A lead to the decrease of the average duration τ of the laminar phase (laminar length), correspondingly the frequency of the jumps to the laminar phase increases [Fig. 4(b,c)], so that finally the laminar phases can no longer be identified. The average laminar length τ also depends on the modulation frequency f because the change in f is equivalent to the change in the velocity at which the control parameter crosses the bifurcation point.

Two series of experiments have been carried out. From all the laminar phases and the sum of the laminar lengths we calculate the average laminar length τ. In the first series we determine τ as a function of A for different fixed f, and in the second series as a function of f for different fixed A. The initial value of the detuning in each series was different and hence the critical amplitude A_c was also different. However, the scaling law is the same and does not depend on the initial value of the detuning. There is no critical value for the modulation frequency. This means that intermittency appears at any $f>0$ as soon as $A>A_c$. In Fig. 5(a) we present the amplitude dependence of the mean laminar length in a log-log scale for fixed $f=73$ Hz. In this experiment $A_c=78.75$ mV. Similar dependence is observed when we fix the modulation amplitude at $A=200$ mV and change the modulation frequency [Fig. 5(b)]. We find that these dependences obey the following power laws:

$$\tau \sim (A-A_c)^{-1}, \tag{1}$$

$$\tau \sim f^{-1}. \tag{2}$$

The diagonal lines in Fig. 5 show a slope of -1.

Both the amplitude and frequency variations are equivalent to a change in the velocity at which the control parameter (detuning) crosses the bifurcation point (onset of intermittency). Thus, the scaling relations Eqs. (1) and (2) can be rewritten as a single relation,

$$\tau \sim v^{-1}, \tag{3}$$

where v is the velocity at which the control parameter is varied.

Similar intermittent behavior is also observed by modulating the injection current near the onset of intermittency. For this purpose we apply a triangular-shape signal to the bias *T*. In this case both laser frequencies remain fixed, however, their gain factors become time dependent. We find that the pump modulation displays the same power-law dependences Eqs. (1)–(3) with the same scaling exponent of -1. Our experimental results agree with theoretical predictions of the on-off intermittency when the control parameter is driven randomly [8]. This agreement verifies that the phenomenon we observed experimentally is the on-off intermittency. According to the theory of the two-state on-off intermittency [11], the external periodical modulation is the reason for the intermittent behavior.

In conclusion, the phenomenon of two-state on-off intermittency has been observed experimentally for the first time, to the best of our knowledge. We have applied a periodic modulation to a control parameter and measured the power-law scaling of the mean laminar phase as a function of both the modulation frequency and amplitude control near the onset. We have found a critical exponent of -1 that coincides with the power law theoretically expected for the on-off intermittency in the case of a random driving. The latter fact verifies a universal character of this scaling relation for different types of driving modulation and different types of the on-off intermittencies. In experiments one knows the driving signal well and hence the appropriate modulation parameters can be computed and applied to the system in order to control the mean laminar phase even without the knowledge of an adequate theoretical model.

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